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An analysis of several novel frameworks and models in the consensus reaching process

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Abstract

Usually, at the beginning of the group decision making (GDM) problem, experts' opinions may differ substantially. Therefore, the consensus reaching process is often a necessity in GDM, and numerous approaches for modeling the consensus process have been proposed. This paper provides an analysis for several novel consensus frameworks and models, investigated by our group. They are the consensus models with minimum adjustments, the consensus models based on consistency and consensus measures, and the direct consensus framework for GDM with different preference representation structures. The advantages of these consensus frameworks and models are analyzed. Meanwhile, the drawbacks and future researches are discussed.

Keywords: Group decision making; consensus; consistency; minimum adjustments; preference representation structures.

1. Introduction

Group decision making (GDM) can be seen as a task to find a collective solution to a decision problem in situations where a group of experts express their opinions regarding multiple alternatives[5]. In general, there are two processes to implement before obtain a final solution[18,22], namely: (i) the selection process; and (ii) the consensus

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process. The selection process obtains the final solution according to the preferences given by the experts. The consensus process involves maximizing consensus or agreement among a group of experts.

Consensus reaching process is a key issue in GDM. Classically, consensus is defined as the full and unanimous agreement of all the experts regarding all the alternatives. However, this definition is inconvenient, and a complete agreement is not always necessary in practice. This has led to use of different consensus measures[25], and numerous approaches for modeling the consensus reaching process have been presented[18,22,24,29]. Cabrerizo et al.[6] and Herrera-Viedma et al.[20] gave excellent surveys of consensus models.

Recently, several novel frameworks and models of the consensus reaching process are proposed.

(1) Consensus models with minimum adjustments. In consensus reaching process, the feedback adjustment rules are often used to help experts adjust their opinions in order to reach a consensus. A natural question is how to minimize the adjustments amounts, which reflects the deviation between experts' original opinions and adjusted opinions. To do so, several consensus models[3,4,14,33,35,37] have been presented.

(2) Consensus models based on consistency and consensus measures. There are two kinds of measures in GDM with preference relations[19,23]: (i) individual consistency, and (ii) consensus. The individual consistency is performed to ensure the expert is being neither random nor illogical in his/her pairwise comparisons, and the consensus means the preferences among a group of experts are similar. In consensus reaching process, the individual consistency may be destroyed. To maintain individual consistency in consensus reaching process, several approaches[12,16,34,36] have been proposed.

(3) Direct consensus framework for GDM with different preference representation structures. In GDM problems, the experts may use different preference representation structures to express their individual preference information, due to different experience, cultures and educational backgrounds. Using transformation functions[7,8,9,22] to uniform different preference representation structures may cause internal inconsistency issues. To avoid inconsistency issue, a direct consensus framework is proposed by Dong and Zhang[15], meanwhile the Pareto principle of social choice theory is satisfied.

The aim of this paper is to analyze these novel consensus frameworks and models. The advantages of these consensus frameworks and models are pointed out. Meanwhile, the drawbacks and future researches are discussed.

The rest of this paper is organized as follows. Section 2 introduces the consensus models with minimum adjustments. Following this, the consensus models based on consistency and consensus measures are presented in Section 3. Subsequently, the direct framework for GDM with different preference representation structures is introduced in Section 4. Finally, Section 5 analyzes the advantages, drawbacks and future researches.

2. Consensus models with minimum adjustments

Let $E=\{e_1,e_2,...,e_m\}$ be a set of m experts. Let $o_k \in R$ and $\overline{o}_k \in R$ represent the original and adjusted preferences of the expert $e_k \in E$, respectively. And the original and adjusted collective preferences are denoted as o and $\overline{o}$, respectively.

The key issue in consensus reaching process is to obtain the $\overline{o}_k$ $(k=1,2,...,m)$ and $\overline{o}$ with minimum adjustments amounts. To do so, two versions of minimum adjustments consensus models are proposed. One of these two versions seeks to minimize the distance between the original and adjusted preferences[37], and the other one seeks to minimize number of adjusted preference values[33].

2.1. Minimizing the distance between the original and adjusted preferences

If $|o_k - o| \le \alpha$, for all $k=1,2,...,m$, the expert opinions reach acceptable consensus, where α is the predefined consensus threshold. For convenience, the threshold of consensus throughout this paper denote as α, which is set according to actual situations. To minimize the distance between the original and adjusted preferences for all experts, Zhang, Dong, Xu and Li[37] proposed an optimization consensus model as follows:

$$\begin{cases} \min_{\overline{o}_k} \sum_{k=1}^{m} |\overline{o}_k - o_k| \\ s.t.\ \overline{o} = Ag(\overline{o}_1, \overline{o}_2, ..., \overline{o}_m) \\ \quad |\overline{o}_k - \overline{o}| \leq \alpha,\ k = 1, 2, ..., m. \end{cases} \tag{1}$$

where, Ag is an aggregation operator. Denote model (1) as M_1. When Ag is weight averaging (WA) operator[31] or ordered weighted averaging (OWA) operators[32], Zhang et al.[37] have shown that M_1 can be transformed into linear programming model. Solving M_1 can yield the $\overline{o}_k$ $(k = 1, 2, ..., m)$ and $\overline{o}$.

2.2. Minimizing the number of adjusted preference values

Let co_k denote a 0-1 variable to count the number of adjusted preference values with respect to the expert $e_k \in E$. If o_k changes in the consensus process, $co_k = 1$; otherwise, $co_k = 0$. By minimizing the number of adjusted preference values, Zhang and Dong[33] proposed an optimization consensus model. For convenience, considering signal attribute and alternative in Zhang and Dong' model, as follows:

$$\begin{cases} \min \sum_{k=1}^{m} co_k \\ s.t.\ \left(\sum_{k=1}^{m} |\overline{o}_k - \overline{o}|\right) \leq \alpha \\ \quad \overline{o} = Ag(\overline{o}_1, \overline{o}_2, ..., \overline{o}_m) \\ \quad co_{ij}^{k} = \begin{cases} 0, & o_k = \overline{o}_k \\ 1, & o_k = \overline{o}_k \end{cases} \end{cases} \tag{2}$$

where, $(\sum_{k=1}^{m} |\overline{o}_k - \overline{o}|) \leq \alpha$ guarantee the predefined consensus among experts are reached. Denote model (2) as M_2. When Ag is OWA operator, Zhang and Dong[33] have shown that M_2 can be equivalently transformed into mixed 0-1 programming model. Solving M_2 can yield the $\overline{o}_k$ $(k = 1, 2, ..., m)$ and $\overline{o}$.

Note1: The detailed solving process of models M_1 and M_2 can be found in Zhang et al.[37] and Zhang[33], respectively. The extended versions of M_1 and M_2 can be found in Zhang, Dong and Xu[35]. Further, M_1 and M_2 models can be extend to modeling the GDM with preference relations.

3. Consensus models based on consistency and consensus measures

Let $E = \{e_1, e_2, ..., e_m\}$ be as before, and let $X = \{x_1, x_2, ..., x_n\}$ be a set of n alternatives. Assume that experts express their opinions over X using preference relations. The decision problem is how to obtain the consensus solution, meanwhile acceptable consistency of individual preference relation are obtained.

Herrera et al.[19] and Chiclana et al.[12] proposed the consensus framework for integrating individual consistency measure (see Fig. 1). The implementation of the framework deals with a two-step procedure.

(1) Consistency improving process. Before each round of consensus reaching process, the consistency control method is used to help the experts obtain the preferences with acceptably consistency.

(2) Consensus improving process. Once all preferences are of acceptably individual consistency, the consensus improving process is applied to help experts to modify their preferences.

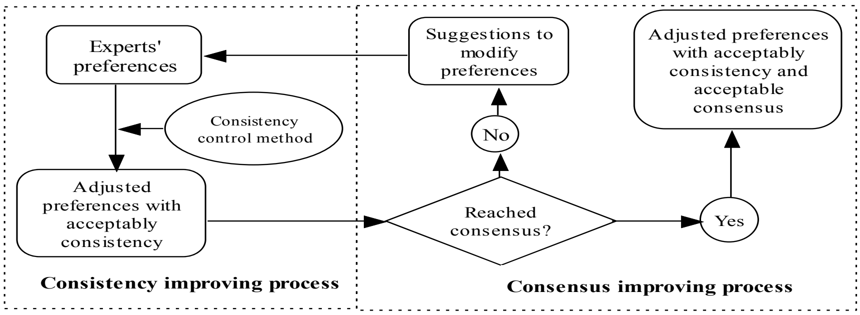

Fig. 1. Herrera et al. and Chiclana et al.' consensus framework.

Repeat these two processes, until the adjusted preferences with acceptably consistency and acceptable consensus are achieved.

Recently, by incorporating consistency and consensus measures into one phase, Dong, Zhang, Hong and Xu[16] and Zhang, Dong and Xu[34] proposed the novel consensus framework (see Fig. 2).

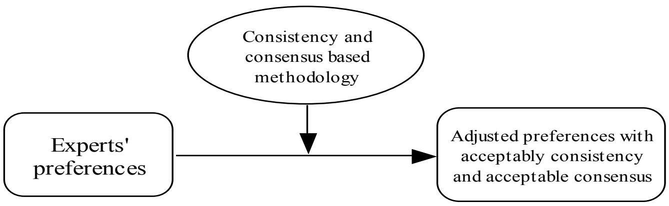

Fig. 2. Dong et al.[16] and Zhang et al.[34]' consensus framework.

Specifically, Dong et al.[16] proposed the iteration consensus model to address GDM with multiplicative preference relations, and Zhang et al.[34] presented the optimization consensus model to address GDM with additive preference relations (i.e., fuzzy preference relations). When using Dong et al.' model, the individual consistency is maintained in the consensus reaching process. And, in Zhang et al.' optimization consensus model, both the acceptable individual consistency and consensus are obtained at the same time. Due to the multiplicative preference relations and additive preference relations can transform into each other[8,22], the research results under one preference relations may be applied in other preference relations.

3.1. Dong, Zhang, Hong and Xu[1]' consensus model for GDM with multiplicative preference relations

Assume that experts using multiplicative preference relations to express their preferences. This subsection introduces the Dong et al.[16]' consensus model, which is denoted as M_3. It deals with a two-step procedure.

(1) Consistency and consensus measures

Let $A^k=(a_{ij}^k)_{n\times n}$ be a multiplicative preference relation provided by the expert $e_k \in E$, where $a_{ij}^k>0$ represents a ratio of the preference intensity of alternative x_i to that of x_j, and $a_{ij}^k \times a_{ji}^k = 1$. Let $P^k=(p_1^k, p_2^k, ..., p_n^k)^T$ be the individual priority vector derived from A^k. Let $P^c=(p_1^c,p_2^c,...,p_n^c)^T$ be the collective priority vector derived from $\{A^1,A^2,...,A^m\}$.

The consistency level of A^k is given by $\overline{CL}(A^k)=\frac{2}{(n-1)(n-2)}\sum_{i<j}(\log(a_{ij}^k)-\log(p_i^k)+\log(p_j^k))^2$. When $\overline{CL}(A^k)\le\beta$, the A^k is of acceptably individual consistency, where β is the established threshold. For convenience, the threshold of individual consistency throughout this paper denote as β.

The consensus level of A^k is defined as $CL(A^k)=\frac{2}{(n-1)(n-2)}\sum_{i<j}(\log(a_{ij}^k)-\log(p_i^c)+\log(p_j^c))^2$. If $\forall k$, $CL(A^k)\le\alpha$, the acceptably consensus are reached among the experts.

(2) Feedback adjustment

Let $\overline{A}^k=(\overline{a}_{ij}^k)_{n\times n}$ be the adjusted multiplicative preference relation associated with e_k. Without loss of generality, suppose that A^τ has a largest consensus level value. When constructing $\overline{A}^k$, we suggest that

$$\overline{a}_{ij}^k=\begin{cases} a_{ij}^k, & k\ne\tau \\ (a_{ij}^k)^\theta(p_i^c/p_j^c)^{(1-\theta)}, & k=\tau \end{cases} \tag{3}$$

where $0<\theta<1$.

Repeat the procedures of consistency and consensus measures and feedback adjustment, until all the multiplicative preference relations with acceptable consensus are obtained.

3.2. Zhang, Dong and Xu[34]' consensus model for GDM with additive preference relations

Assume that experts using additive preference relations to express their opinions over the alternatives. This section introduces the consensus model proposed by Zhang et al.[34].

Let $\{F^1,...,F^m\}$ be a group of additive preference relations, where $F^k=(f_{ij}^k)_{n\times n}$ is provided by the expert e_k, $f_{ij}^k\in[0,1]$ denotes the preference degree of the alternative x_i over x_j, and $f_{ij}^k+f_{ji}^k=1$. The key task of reaching consensus among $\{F^1,...,F^m\}$ is to find a group of individual additive preference relations $\{\overline{F}^1,...,\overline{F}^m\}$ with acceptable consistency and consensus, where $\overline{F}^k=(\overline{f}_{ij}^k)_{n\times n}$.

The consistency level of F^k is defined as $\overline{CL}(F^k)=\frac{2}{3n(n-1)(n-2)}\sum_{i,t=1;i\ne t}^{n}\sum_{j=1;j\ne i,t}^{n}|f_{ij}^k+f_{jt}^k-f_{it}^k-0.5|$. If $\overline{CL}(F^k)\le\beta$, the F^k is of acceptably individual consistency.

Consensus level among $\{F^1,...,F^m\}$ is defined as $CL\{F^1,...,F^m\}=\frac{2}{nm(m-1)(n-1)}\sum_{i=1}^{n}\sum_{j=1,j\ne i}^{n}\sum_{t\ge r}^{m}\sum_{r=1}^{m}|f_{ij}^r-f_{ij}^t|$. If $CL\{F^1,...,F^m\}\le\alpha$, the acceptably consensus is reached among the experts.

In order to obtain $\{\overline{F}^1,...,\overline{F}^m\}$ with acceptable consistency and consensus, and to preserve the information $\{F^1,...,F^m\}$ as much as possible, an optimization model to reaching consensus is constructed as follows:

$$\begin{cases} \min_{\overline{F}^k}\sum_{k=1}^m d(F^k,\overline{F}^k) \\ s.t.\ \overline{CL}(\overline{F}^k)\le\beta,\ k=1,2,...,m \\ \qquad CL\{\overline{F}^1,...,\overline{F}^m\}\le\alpha \end{cases} \tag{4}$$

where, $d(F^k,\overline{F}^k)=\frac{1}{n^2}\sum_{i=1}^n\sum_{j=1}^n|f_{ij}^k-\overline{f}_{ij}^k|$ is distance between F^k and $\overline{F}^k$, $\overline{CL}(\overline{F}^k)\le\beta$ guarantee $\overline{F}^k$ has acceptably consistency, and $CL\{\overline{F}^1,...,\overline{F}^m\}\le\alpha$ guarantee $\{\overline{F}^1,...,\overline{F}^m\}$ has acceptable consensus.

Denote model (4) as M_4. Solving M_4 can obtain the $\overline{F}^k$ $(k=1,2,...,m)$.

Note 2: To manage the incomplete additive preference relations in GDM, several approaches have been proposed[10,17,21]. Chiclana et al.[11] investigated the improved method to measure the consistency of the additive preference relations.

4. Direct consensus framework for GDM with different preference representation structures

Suppose experts' preference information over the alternatives may be represented in one of the four formats, i.e., preference orderings, utility functions, multiplicative preference relations, and additive preference relations. The decision problem is how to obtain the ranking of the alternatives with acceptable consensus.

Chinala et al.[7] initiated an indirect model to rank the alternatives, and Herrera-Viedma et al.[22] presented the corresponding consensus model. Inspired by these two models, Dong and Zhang[15] proposed the direct consensus framework (see Fig. 3), which is denoted as M_5. In the direct consensus framework, the selection process and the consensus process are used.

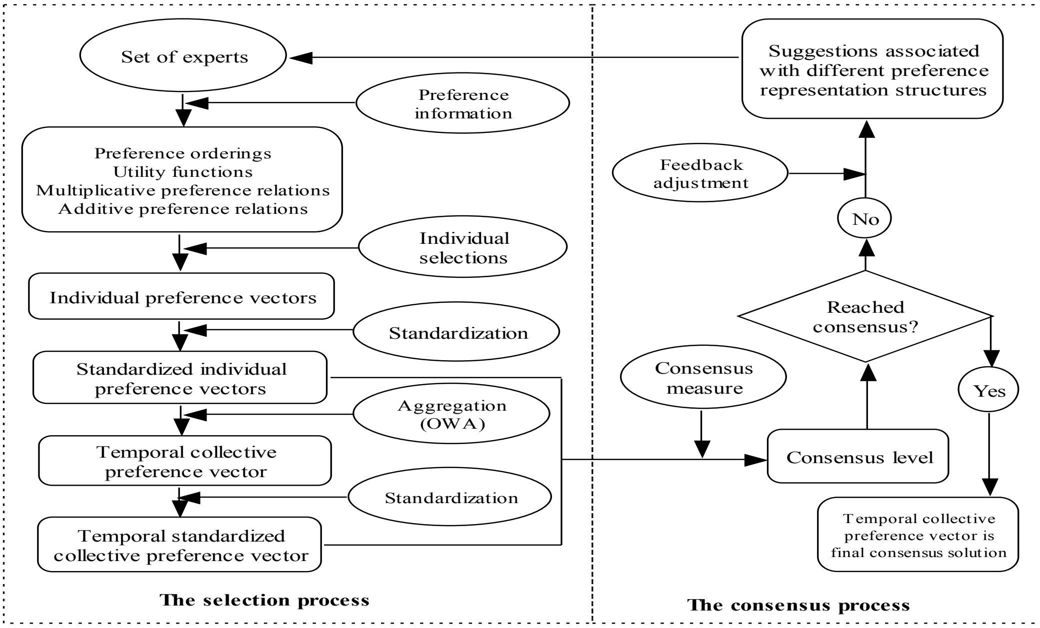

Fig.3. The direct framework of GDM problems.

(1) Selection process

Let E^U, E^{Po}, E^A and E^F be four subsets of E, representing experts whose preference information on X are expressed by utility functions, preference orderings, multiplicative preference relations, and additive preference relations, respectively

(i) Obtaining the individual preference vectors

To obtain the individual preference vectors $P^k=(p_1^k, p_2^k, ..., p_n^k)^T$ $(k=1,2,...,m)$, four cases are considered.

Case A: $e_k \in E^U$, i.e., the expert e_k using utility function to express his/her opinion, $U^k=(u_1^k,...,u_n^k)^T$, where $u_i^k \in [0,1]$ is the utility value of the alternative x_i. In this case, $p_i^k = u_i^k$.

Case B: $e_k \in E^{Po}$, i.e., the expert e_k using preference ordering to express his/her opinions, $Po^k=(po_1^k,...,po_n^k)^T$, where Po_i^k is the position of the alternative x_i in $\{x_1,...,x_n\}$. In this case, $p_i^k=(n-po_i^k)/(n-1)$.

Case C: $e_k \in E^A$, i.e., the expert e_k using multiplicative preference relation to express his/her opinions, $A^k=(a_{ij}^k)_{n\times n}$. In this case, $p_i^k = (\prod_{j=1}^n a_{ij}^k)^{1/n} \Big/ \sum_{i=1}^n (\prod_{j=1}^n a_{ij}^k)^{1/n}$.

Case D: $e_k \in E^F$, i.e., the expert e_k using additive preference relation to express his/her opinions, $F^k=(f_{ij}^k)_{n\times n}$. In this case, $p_i^k = OWA(f_{i1}^k, f_{i2}^k, ..., f_{in}^k)$.

(ii) Obtaining a collective preference vector

Transform $P^k = (p_1^k,...,p_n^k)^T$ $(k = 1,2,...,m)$ into the standardized individual preference vector $P^{k*} = (p_1^{k*},...,p_n^{k*})^T$, where $p_i^{k*} = p_i^k \Big/ \sum_{i=1}^n p_i^k$. Then, a collective preference vector $P^c = (p_1^c, p_2^c,..., p_n^c)^T$ is obtained, where $p_i^c = OWA(p_i^{1*},..., p_i^{m*})$. Normalizing P^c yields the standardized collective preference vector $P^{c*} = (p_1^{c*},..., p_n^{c*})^T$, where $p_i^{c*} = p_i^c \Big/ \sum_{i=1}^n p_i^c$.

(2) Consensus process

(i) Consensus measure.

Let P^{k*} and P^{c*} be as earlier. The consensus level for e_k is defined as $CL(e_k) = \sqrt{\frac{1}{n}\sum_{i=1}^n (p_i^{k*} - p_i^{c*})^2}$.

The consensus level among all experts is defined as $CL\{e_1,...,e_m\} = \frac{1}{m}\sum_{k=1}^m CL(e_k)$. Obviously, the smaller $CL\{e_1,...,e_m\}$ value indicates the higher consensus among experts.

(ii) Feedback adjustment

The feedback adjustment rules, consist of two steps: 1) Transforming the standardized collective preference vector P^{c*} into the preference information that is described by the preference representation structure used by the individual expert; 2) the experts revised his/her preferences, according to the original individual preference information and the transformed collective preference information. According to the formats of preference representation structures, four cases for feedback adjustment are considered.

Case A: $e_k \in E^U$. Transforming P^{c*} into $U^{c,k} = (u_1^{c,k},...,u_n^{c,k})^T$, where $u_i^{c,k} = p_i^{c*}\sum_{i=1}^n u_i^k$. Let $\bar{U}^k = (\bar{u}_1^k,...,\bar{u}_n^k)^T$ be the adjusted utility function provided by e_k. When constructing $\bar{U}^k$, we suggest that

$$\bar{u}_i^k \in [\min(u_i^k, u_i^{c,k}), \max(u_i^k, u_i^{c,k})]. \tag{5}$$

Case B: $e_k \in E^{Po}$. Let $Po^{c,k} = (po_1^{c,k},..., po_n^{c,k})^T$ be the transformed collective preference information described by preference orderings. If po_i^{c*} is t th largest value in $\{p_1^{c*},..., p_n^{c*}\}$, then $po_i^{c,k} = t$. Let $\overline{Po}^k = (\overline{po}_1^k,...,\overline{po}_n^k)^T$ be the adjusted preference ordering provided by e_k. When constructing $\overline{Po}^k$, we suggest that

$$\overline{po}_i^k \in [\min(po_i^k, po_i^{c,k}), \max(po_i^k, po_i^{c,k})]. \tag{6}$$

Case C: $e_k \in E^A$. Transforming P^{c*} into a multiplicative preference relation $A^{c,k} = (a_{ij}^{c,k})_{n\times n}$, where $a_{ij}^{c,k} = p_i^{c*} \big/ p_j^{c*}$. Let $\bar{A}^k = (\bar{a}_{ij}^k)_{n\times n}$ be the adjusted multiplicative preference relation provided by e_k. When constructing $\bar{A}^k$, we suggest that

$$\begin{cases} \bar{a}_{ij}^k \in [\min(a_{ij}^k, a_{ij}^{c,k}), \max(a_{ij}^k, a_{ij}^{c,k})], & i \le j \\ \bar{a}_{ij}^k = 1 \big/ \bar{a}_{ji}^k, & i > j \end{cases}. \tag{7}$$

Case D: $e_k \in E^F$. Transforming P^{c*} into a additive preference relation $F^{c,k} = (f_{ij}^{c,k})_{n\times n}$, where $f_{ij}^{c,k} = p_i^{c*} \big/ (p_i^{c*} + p_j^{c*})$. Let $\bar{F}^k = (\bar{f}_{ij}^k)_{n\times n}$ be the adjusted additive preference relation provided by e_k. When constructing $\bar{F}^k$, we suggest that

$$\begin{cases} \bar{f}_{ij}^k \in [\min(f_{ij}^k, f_{ij}^{c,k}), \max(f_{ij}^k, f_{ij}^{c,k})], & i \le j \\ \bar{f}_{ij}^k = 1 - \bar{f}_{ji}^k, & i > j \end{cases}. \tag{8}$$

Repeated the selection process and consensus process until the adjusted preference information with established consensus level are obtained.

5. Advantages, drawbacks and future researches

This section analyzes the advantages of the novel consensus frameworks and models introduced above, meanwhile, the drawbacks and future researches are discussed.

5.1. Advantages

(1) Consensus models with minimum adjustments

This kind of consensus models has ability to minimize the distance between the original and adjusted preferences, or the number of adjusted preference values. In other words, the original preference information is preserved as many as possible in consensus reaching process. Compared with the consensus models presented in Ben-Arieh and Easton[3] and Ben-Arieh et al.[4], the aggregation operators are incorporated in M_1, M_2.

(2) Consensus models based on consistency and consensus measures

In consensus frameworks[12,19], two processes are used to improve the consistency and consensus (see Fig. 1). In M_3, the individual consistency is maintained in the consensus reaching process, and some desired properties are satisfied: (i) the adjusted multiplicative preference relations has a better consistency than the corresponding original multiplicative preference relations; and (ii) it satisfies the Pareto principle of social choice theory[2]. And, in M_4, both the acceptable consistency and consensus are obtained in an optimization model, meanwhile the original preference information is optimally preserved.

(3) Direct consensus framework for GDM with different preference representation structures

The model M_5 satisfies two desirable properties: (i) the proposed framework can avoid internal inconsistency issue when using the transformation functions among different preference representation structures[9]; (ii) it satisfies the Pareto principle of social choice theory[2].

5.2. Drawbacks and future researches

(1) The model M_2 is transformed into mixed 0-1 programming problem. However, it is very complex and difficult to solve linear and mixed 0-1 linear programming problems, when the number of variables is large. In future researches, it is very interesting to design the effective approaches for solving consensus model M_2.

(2) The consensus models M_1, M_2, M_3 and M_4, should only be considered as a decision aid which experts use as a reference to modify their individual preference information. So, an interesting and promising research topic is to study the interactive consensus reaching process based on the guidance given by these four consensus models.

(3) The behaviors and attitudes[26,27] of the experts are not considered in these novel consensus models and frameworks introduced above. Thus, an interesting and promising research topic is to incorporate the behaviors and attitudes of experts into the consensus models, and thus provide a flexible framework to constitute a better approximate decision model to real-world GDM problems.

(4) Most of the existing consensus frameworks and consensus models are not considering dynamic situations. We argue that it will be interesting in future research to investigate the consensus reaching model in dynamic situations[1,28], in which the participation and contribution rates of experts and feasible alternatives are dynamic changed.

(5) For facilitating consensus reaching process, a large number of consensus frameworks and models have been developed. Recently, Chiclana et al.[13] presented a comparative study of the application of different functions for measuring consensus in GDM. However, there lack of a common framework and criteria to evaluate different consensus models and consensus frameworks. Therefore, it is interesting in any future research to investigate the comparison framework to compare different consensus models.

(6) In the three main methodological approaches to consensus, the preferences are using crisp numerical values (in [0, 1] or [1/9, 9]). It is interesting in future research to study the consensus models with uncertain preferences relations. The consensus model presented by Wu and Chiclana[30] is a good example to conduct such study.

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